

Lecture 08/31/23 Domain and Range

Interval Notation: When describing domain and range it is ~~convinient~~ convenient to have a short hand for describing large sets of numbers.

$$[a, b] \quad \longleftrightarrow \quad a \leq x \leq b$$

$$[a, b) \quad \longleftrightarrow \quad a \leq x < b$$

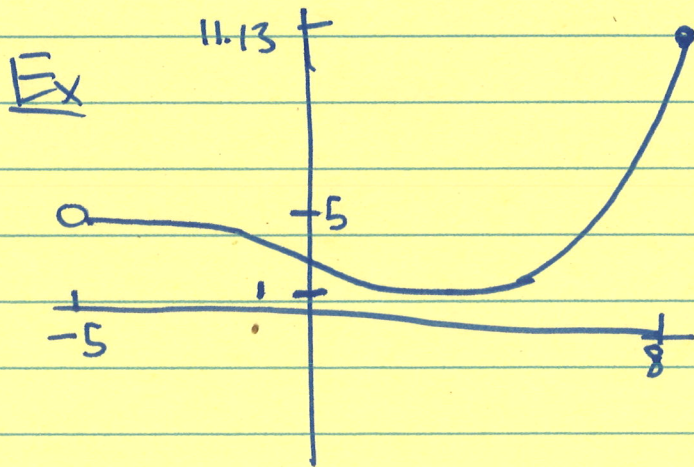
$$(a, b] \quad \longleftrightarrow \quad a < x \leq b$$

$$(a, b) \quad \longleftrightarrow \quad a < x < b$$

Recall: The domain of a function is the set of all x -values that are allowed to go into the function. The range of a function is the set of all y -values that are allowed to go into the function.

Ex

Domain + Range from Graphs:



$$D: [-5, 8]$$

$$R: [1, 11.13]$$

Domain + Range from equations

Ex: Find domain and range of

$$g(x) = \frac{1}{\sqrt{x^2 - 25}}$$

We can't plug negative #'s into square root, so we need to see when ~~the denominator is positive~~ $x^2 - 25$ is positive. This is the same thing as seeing when

$$\begin{aligned} x^2 - 25 &\geq 0 \\ \iff \\ x^2 &\geq 25 \\ \iff \\ -5 &\leq x \leq 5 \quad \text{or} \quad x \geq 5 \quad \text{or} \quad x \leq -5 \\ \downarrow & \qquad \qquad \qquad \downarrow \\ (-\infty, -5] & \qquad \qquad \qquad [5, \infty) \end{aligned}$$

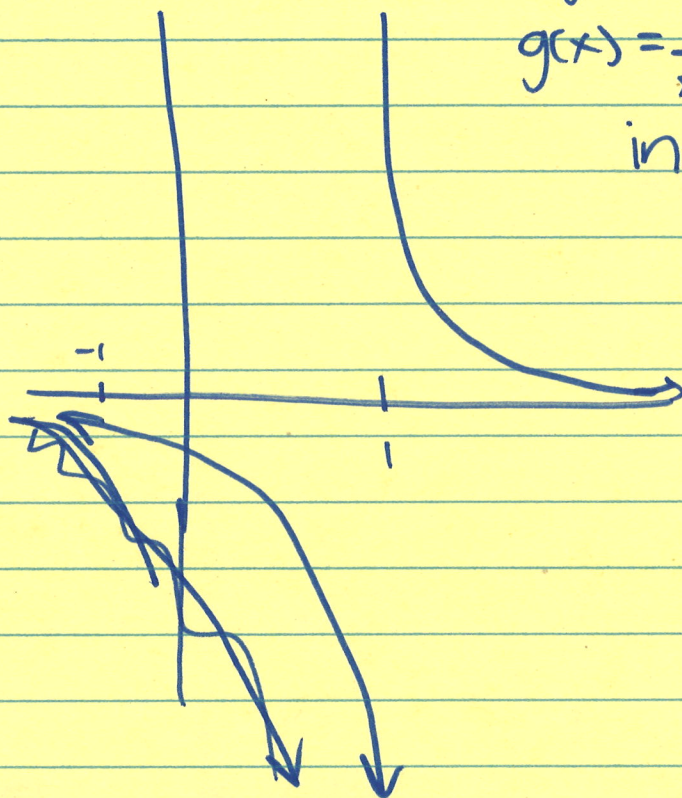
The denominator can't be zero. This happens when $x = -5, 5$ so we must remove 5 and -5 from above. Hence domain is $(-\infty, -5] \cup [5, \infty)$

To find range use a graphing calc. Desmos demonstration.

Restricting Domain: Sometimes we will only look at pieces of a function by shortening the domain.

Ex: Determine the range of the function

$$g(x) = \frac{1}{x-1} \quad \text{on the interval } (-1, 2)$$



$$g(-1) = \frac{1}{-1-1} = -\frac{1}{2} \quad g(x) \text{ travels to}$$

$-\infty$ as x approaches 1. Then, it comes from ∞ until it hits $g(2) = \frac{1}{2-1} = \frac{1}{2}$ so range is

$$\left(-\infty, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$$